

水下砂土层非线性动力反应的直接差分法

陈文化

(北京交通大学 土木建筑学院, 北京 100044)

摘要: 应用多孔介质动力学进行了水下的砂土地层的动力反应分析, 水下砂层可以看作是多相介质, 孔隙水是可压缩的, 土壤采用了非线性本构, 考虑了砂土的刚度退化和剪胀性及土水相对运动等因素, 并研究了多孔介质波动模拟的边界条件, 建立了水体下地层平面动力非线性动力反应的直接差分法, 直接求解水体的压力和饱和土层的动孔隙压力, 可以进一步分析水下砂土地震液化问题。

关键词: 砂土; 多相介质动力学; 直接差分法

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A direct differential method for nonlinear dynamic response of sand layer under water

CHEN Wen-hua

(Civil Engineering Department, Beijing Jiaotong University, Beijing 100044, China)

Abstract: Based on K. Terzaghi's soil mechanics model and taken account of soil dynamic nonlinear constitutive relation, soil shear dilatancy, rigidity degradation, hysteresis and water-soil relative motion, multi-phased medium dynamics theory is used to analyze the wave propagation and dynamic response of sand layers under water, and an artificial boundary condition is analyzed to simulate the wave propagation in pore medium. A set of dynamic equations of soil layers under water are established, and the equations are discretized to be used to analyze the earthquake engineering problem of soil under water, a direct differential method is suggested to calculate the water pressure and porewater pressure and to study the sand seismic liquefaction under water.

Key words: sand; multi-phased medium dynamics; direct differential method

1 引言

有散射场的动力问题是岩土工程、地震工程经常讨论的课题。在土层动力反应分析中, 自由场地不受散射场的影响, 而非自由场地则不同, 在动力作用下既可能有散射场的作用, 而且又受到结构体系的影响, 存在动力相互作用。非自由场地大致可分为3类: 第1类为无形散射源, 如地形条件的影响, 坡形或凹凸地形在动力反应时存在散射作用。第2类为有形散射源, 如结构物、地形条件对于土层动力反应影响, 经典的问题是如何求解弹性半空间半圆弧(凹凸地形)的散射波的解析解或数值解, 有了一些研究结果^[1], 这些研究工作无疑对于提高问题的认识产生积极的作用, 但这些研究工作前提假设认为, 土层或

地层为单一介质体系, 动力反应过程中土壤是弹性的, 或者非线性问题线性化, 目前主要采用数值模拟^[2]。本文主要讨论水体下(第3类散射)动力反应数值直接差分法。

2 动力反应理论

2.1 平衡方程

水体的动力方程为

$$\left. \begin{aligned} \frac{\partial \sigma_{lq,x}}{\partial x} &= \frac{\partial^2 u_x}{\partial t^2} \\ \frac{\partial \sigma_{lq,z}}{\partial y} &= \frac{\partial^2 u_y}{\partial t^2} \end{aligned} \right\} \quad (1)$$

饱和地基的动力方程为

(1) 饱和土固相:

$$\left. \begin{aligned} & \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + (1-n) \frac{\partial \sigma_w}{\partial x} + \\ & b_x \left(\frac{\partial v_x}{\partial t} - \frac{\partial u_x}{\partial t} \right) = (1-n) \rho_s \frac{\partial^2 u_x}{\partial t^2} \\ & \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + (1-n) \frac{\partial \sigma_w}{\partial y} + \\ & b_y \left(\frac{\partial v_y}{\partial t} - \frac{\partial u_y}{\partial t} \right) = (1-n) \rho_s \frac{\partial^2 u_y}{\partial t^2} \end{aligned} \right\} \quad (2)$$

(2) 液相:

$$\left. \begin{aligned} & n \frac{\partial \sigma_w}{\partial x} + b_x \left(\frac{\partial u_x}{\partial t} - \frac{\partial v_x}{\partial t} \right) = n \rho_w \frac{\partial^2 v_x}{\partial t^2} \\ & n \frac{\partial \sigma_w}{\partial y} + b_y \left(\frac{\partial u_y}{\partial t} - \frac{\partial v_y}{\partial t} \right) = n \rho_w \frac{\partial^2 v_y}{\partial t^2} \end{aligned} \right\} \quad (3)$$

式中: u_x, u_y 分别为土骨架 x, y 方向的位移; v_x, v_y 分别为水 x, y 方向的位移; ρ_s, ρ_w 分别为土骨架和水的密度; b_x, b_y 分别为 x, y 方向的渗透阻尼系数, 渗透性在空间上有较大差异, 垂直方向一般比水平方向大, 此处假定沿水平方向渗透性一致; σ_x, σ_y 均为有效应力; σ_w 为孔隙水压力; τ_{xy}, τ_{xz} 均为剪应力; n 为孔隙率。

2.2 连续方程

$$n \frac{\partial \bar{\varepsilon}_v}{\partial t} + (1-n) \frac{\partial \bar{\varepsilon}_u}{\partial t} = \frac{n}{E_w} \frac{\partial \sigma_w}{\partial t} \quad (4)$$

式中: E_w 为水的压缩模量; $\bar{\varepsilon}_v$ 为水体的压缩体应变; $\bar{\varepsilon}_u$ 为土骨架的体应变。

2.3 本构关系

(1) 剪切本构关系

$$\tau = g(\sigma) f(\gamma) \quad (5)$$

(2) 体积变化的本构关系

$$\frac{\partial \varepsilon_{ad}}{\partial t} + \frac{1}{E_k} \frac{\partial}{\partial t} \left(\frac{\sigma_x + \sigma_y + \sigma_z}{3} \right) + \frac{1-n}{E_s} \frac{\partial \sigma_w}{\partial t} = \frac{\partial \bar{\varepsilon}_u}{\partial t} \quad (6)$$

式中: E_k, E_s 分别为土颗粒和土骨架的弹性模量; ε_{ad} 为剪胀体变。

3 土的动力本构关系

3.1 Masing 模型^[3]

Masing^[3]提出的基于骨架曲线的应力-应变

关系, 其骨架曲线的方程式可写成:

$$\tau = f(\gamma) = \frac{G\gamma}{1 + \gamma/\gamma_r} \quad (7)$$

卸载及反向加载过程中, 初始卸载模量与土体最大动模量相等, 且应力-应变关系与骨架曲线成“2 倍法”:

$$\tau - \tau_c/2 = f(\gamma - \gamma_c/2) \quad (8)$$

式中: τ_c, γ_c 分别为应力-应变曲线最近一个拐点的剪应力、剪应变值。此模型仅适用于等幅荷载。

3.2 体积变化模型

大量干砂的剪胀附加体应变的全量随循环次数开始时增加较快, 几个循环后便慢多了。尽管不同的砂、不同的剪应力幅值, 曲线的弯曲度不一样, 但曲线对应的曲线类型相似, 当剪应变幅值一定时, 体积应变随往返作用次数增加而增加。因此, 文献[5]研究了减缩体变与剪应力(或剪应变)之间的关系, 针对体变全量随循环次数增多而逐渐减小, 但各循环 $\Delta \varepsilon_{adv}$ 的累加作为时间 t 的函数, 则是逐渐增加的, 不过后来增加得越来越少, 把剪胀体应变的全量与时间的关系写成下列简便的表达式:

$$\varepsilon_{ad} = a(1 - e^{-bt}) \quad (9)$$

式中: a 与土性有关参数; b 为荷载有关参数; t 为时间变量。

4 方程的离散

令土骨架和水的节点位移分别为 δ_u, δ_v , 利用分部积分和Green定理, 方程(1)~(6)可离散为

$$\begin{aligned} & [K_1]\{\delta_u\} + [S_1]\{\sigma_w\} + [C_1]\{\dot{\delta}_u - \dot{\delta}_v\} + \\ & [M_1]\{\ddot{\delta}_u\} = \{F(t)_1\} \end{aligned} \quad (10)$$

$$\begin{aligned} & [S_2]\{\sigma_w\} - [C_2]\{\dot{\delta}_u - \dot{\delta}_v\} + \\ & [M_2]\{\ddot{\delta}_v\} = \{F_2(t)\} \end{aligned} \quad (11)$$

$$[A_1]\{\dot{\delta}_u\} + [A_2]\{\dot{\delta}_v\} + [A_3]\{\dot{\sigma}_w\} = \{0\} \quad (12)$$

其中:

$$[K_1] = \iint \begin{bmatrix} (\lambda + 2G) \frac{\partial N_i \partial N_j}{\partial x \partial x} + G \frac{\partial N_i \partial N_j}{\partial z \partial z} & G \frac{\partial N_i \partial N_j}{\partial y \partial x} + \lambda \frac{\partial N_i \partial N_j}{\partial y \partial z} & G \frac{\partial N_i \partial N_j}{\partial z \partial x} + \lambda \frac{\partial N_i \partial N_j}{\partial z \partial y} \\ G \frac{\partial N_i \partial N_j}{\partial z \partial x} + \lambda \frac{\partial N_i \partial N_j}{\partial x \partial z} & (\lambda + 2G) \frac{\partial N_i \partial N_j}{\partial y \partial y} + G \frac{\partial N_i \partial N_j}{\partial z \partial z} & G \frac{\partial N_i \partial N_j}{\partial x \partial z} + \lambda \frac{\partial N_i \partial N_j}{\partial y \partial z} \\ G \frac{\partial N_i \partial N_j}{\partial z \partial x} + \lambda \frac{\partial N_i \partial N_j}{\partial z \partial y} & G \frac{\partial N_i \partial N_j}{\partial x \partial z} + \lambda \frac{\partial N_i \partial N_j}{\partial y \partial z} & (\lambda + 2G) \frac{\partial N_i \partial N_j}{\partial z \partial z} + G \frac{\partial N_i \partial N_j}{\partial z \partial z} \end{bmatrix} d\Omega \quad (13)$$

$$[S_1] = \iint_{\Omega} \begin{bmatrix} -(1-n)N_i \frac{\partial N_j}{\partial x} \\ -(1-n)N_i \frac{\partial N_j}{\partial y} \\ -(1-n)N_i \frac{\partial N_j}{\partial z} \end{bmatrix} d\Omega \quad (14)$$

$$[S_2] = \iint_{\Omega} \begin{bmatrix} -nN_i \frac{\partial N_j}{\partial x} \\ -nN_i \frac{\partial N_j}{\partial y} \\ -nN_i \frac{\partial N_j}{\partial z} \end{bmatrix} d\Omega \quad (15)$$

$$[C_1] = \iint \begin{bmatrix} -b_x N_i N & 0 & 0 \\ 0 & -b_y N_i N & 0 \\ 0 & 0 & -b_z N_i N \end{bmatrix} d\Omega \quad (16)$$

$$[C_2] = \iint \begin{bmatrix} -b_x N_i N & 0 & 0 \\ 0 & -b_y N_i N & 0 \\ 0 & 0 & -b_z N_i N \end{bmatrix} d\Omega \quad (17)$$

$$[A_1] = \iint [(1-n)N_i \frac{\partial N_j}{\partial x} \quad (1-n)N_i \frac{\partial N_j}{\partial y} \quad (1-n)N_i \frac{\partial N_j}{\partial z}] d\Omega \quad (18)$$

$$[A_2] = \iint [nN_i \frac{\partial N_j}{\partial x} \quad nN_i \frac{\partial N_j}{\partial y} \quad nN_i \frac{\partial N_j}{\partial z}] d\Omega \quad (19)$$

$$[A_3] = \iint [\frac{n}{E_w} N_i] d\Omega \quad (20)$$

$$[M_1] = \iint_{\Omega} \begin{bmatrix} -\rho_1 N_i N_j & 0 & 0 \\ 0 & -\rho_1 N_i N_j & 0 \\ 0 & 0 & -\rho_1 N_i N_j \end{bmatrix} d\Omega \quad (21)$$

$$[M_2] = \iint_{\Omega} \begin{bmatrix} -\rho_2 N_i N_j & 0 & 0 \\ 0 & -\rho_2 N_i N_j & 0 \\ 0 & 0 & -\rho_2 N_i N_j \end{bmatrix} d\Omega \quad (22)$$

$$\{F_1\} = \iint \begin{bmatrix} -\lambda N_i \frac{\partial N_j}{\partial x} \varepsilon_{dvj} \\ -\lambda N_i \frac{\partial N_j}{\partial z} \varepsilon_{dvj} \end{bmatrix} d\Omega + \int_{\Gamma} \begin{bmatrix} N_i (X + (1-n)P_x) \\ N_i (Z + (1-n)P_y) \end{bmatrix} d\Gamma \quad (23)$$

$$\{F_2\} = \iint_{\Omega} \begin{bmatrix} 0 \\ 0 \end{bmatrix} d\Omega + \int_{\Gamma} \begin{bmatrix} N_{ix} n P_x \\ N_{iy} n P_y \end{bmatrix} d\Gamma \quad (24)$$

$$\{F_1\} = \iint \begin{bmatrix} -\lambda N_i \frac{\partial N_j}{\partial x} \varepsilon_{ad,j} \\ -\lambda N_i \frac{\partial N_j}{\partial x} \varepsilon_{adj} \end{bmatrix} d\Omega + \quad (25)$$

$$\int_{\Gamma} \begin{bmatrix} N_i (X + (1-n)P_x) \\ N_i (Z + (1-n)P_z) \end{bmatrix} d\Gamma \quad (26)$$

$$\left. \begin{aligned} X &= \sigma_x n_x + \tau_{xy} n_z \\ Y &= \sigma_y n_y + \tau_{xy} n_y \end{aligned} \right\}$$

$$\left. \begin{aligned} P_x &= \sigma_w n_x \\ P_y &= \sigma_w n_y \end{aligned} \right\} \quad (27)$$

式中: $\sigma_x, \sigma_y, \sigma_z$ 分别为土骨架 X, Y 方向的正应力; τ_{xz} 为剪应力; σ_w 为孔隙水压力; n_x, n_y 分别为边界的外法线对于 x, y 轴方向余弦。

将方程(12)分别加在方程(10)和方程(11)上, 这样一来成为对下述方程独立求解。

$$\begin{aligned} [K_1]\{\delta_u\} + [S_1]\{\sigma_w\} + [C]\{\dot{\delta}_u - \dot{\delta}_v\} + \\ [M_1]\{\ddot{\delta}_u\} + [A_1]\{\dot{\delta}_u\} + [A_2]\{\dot{\delta}_v\} + \\ A_3\{\dot{\sigma}_w\} = \{F_1\} \end{aligned} \quad (28)$$

$$\begin{aligned} [S_2]\{\sigma_w\} - [C]\{\dot{\delta}_u - \dot{\delta}_v\} + [M_2]\{\ddot{\delta}_v\} + \\ [A_1]\{\dot{\delta}_u\} + [A_2]\{\dot{\delta}_v\} + A_3\{\dot{\sigma}_w\} = \{0\} \end{aligned} \quad (29)$$

$$[A_1]\{\dot{\delta}_u\} + [A_2]\{\dot{\delta}_v\} + A_3\{\dot{\sigma}_w\} = \{0\} \quad (30)$$

$$\text{将 } \{\ddot{\delta}_u(t)\} = \frac{1}{2\Delta t} [\{\dot{\delta}_u(t + \Delta t)\} - \{\dot{\delta}_u(t - \Delta t)\}]$$

代入得

$$\begin{aligned} \{\dot{\delta}_u(t + \Delta t)\} = 2\Delta t [M]^{-1} (-\frac{[M_1]}{2\Delta t} \{\dot{\delta}_u(t - \Delta t)\} + \\ [K]\{\delta_u(t)\} + [S_1]\{\sigma_w(t)\} + [C]\{\dot{\delta}_u(t) - \dot{\delta}_v(t)\} + \\ [A_1]\{\dot{\delta}_u(t)\} + [A_2]\{\dot{\delta}_v(t)\} - \frac{n}{E_w} \{\dot{\sigma}_w(t)\} - \{F_1(t)\}) \end{aligned} \quad (31)$$

$$\text{将 } \{\ddot{\delta}_u(t)\} = \frac{2}{\Delta t^2} [\{\delta_u(t + \Delta t)\} - \{\delta_u(t)\}] -$$

$\frac{2}{\Delta t} \{\dot{\delta}_u(t)\}$ 代入式(28)得

$$\begin{aligned} \{\delta_u(t + \Delta t)\} = \frac{\Delta t^2 [M]^{-1}}{2} (-\frac{2[M_1]}{\Delta t^2} \{\delta_u(t)\} - \\ \frac{2[M_1]}{\Delta t} \{\dot{\delta}_u(t)\} + [K]\{\delta_u(t)\} + [S_1]\{\sigma_w(t)\} + \\ [C]\{\dot{\delta}_u(t) - \dot{\delta}_v(t)\} + [A_1]\{\dot{\delta}_u(t)\} + \\ [A_2]\{\dot{\delta}_v(t)\} + \frac{n}{E_w} \{\dot{\sigma}_w(t)\} - \{F_1(t)\}) \end{aligned} \quad (32)$$

$$\begin{aligned} \text{将 } \{\ddot{\delta}_v(t)\} &= \frac{2}{\Delta t^2} [\{\delta_v(t+\Delta t)\} - \{\delta_v(t)\}] - \\ &\frac{2}{\Delta t} \{\dot{\delta}_v(t)\} \text{ 代入式 (29) 得} \\ \{\delta_v(t+\Delta t)\} &= \frac{[M_2]^{-1} \Delta t^2}{2} (-\frac{2[M_2]}{\Delta t^2} \{\delta_v(t)\} - \\ &\frac{2[M_2]}{\Delta t} \{\dot{\delta}_v(t)\} + [S_2] \{\sigma_w(t)\} - \\ &[C] \{\dot{\delta}_u(t) - \dot{\delta}_v(t)\} + [M_2] \{\ddot{\delta}_v(t)\} + \\ &[A_1] \{\dot{\delta}_u(t)\} + [A_2] \{\dot{\delta}_v(t)\} + A_3 \{\dot{\sigma}_w(t)\}) \quad (33) \end{aligned}$$

$$\begin{aligned} \text{将 } \{\ddot{\delta}_v(t)\} &= \frac{1}{2\Delta t} [\{\dot{\delta}_v(t+\Delta t)\} - \{\dot{\delta}_v(t-\Delta t)\}] \\ \text{代入式 (29), 得} \\ \{\dot{\delta}_v(t+\Delta t)\} &= 2[M_2]^{-1} \Delta t (-\frac{2[M_2]}{\Delta t} \{\dot{\delta}_v(t-\Delta t)\} + \\ &[S_2] \{\sigma_w(t)\} - [C] \{\dot{\delta}_u(t) - \dot{\delta}_v(t)\} - [M_2] \{\ddot{\delta}_v(t)\} + \\ &[A_1] \{\dot{\delta}_u(t)\} + [A_2] \{\dot{\delta}_v(t)\} + A_3 \{\dot{\sigma}_w(t)\}) \quad (34) \end{aligned}$$

将 $\{\dot{\sigma}_w(t)\} = \frac{1}{2\Delta t} \{\sigma_w(t+\Delta t) - \sigma_w(t-\Delta t)\}$ 代入方程(29), 将式(32)和式(33)代入方程(31), 得孔隙水压力为

$$\begin{aligned} \{\sigma_w(t+\Delta t)\} &= \frac{2\Delta t E_w}{n} ([K_1] \{\delta_u(t)\} + \\ &[S_1] \{\sigma_w(t)\} + [C] \{\dot{\delta}_u(t) - \dot{\delta}_v(t)\} + \\ &[M_1] \{\ddot{\delta}_u(t)\} + [A_1] \{\dot{\delta}_u(t)\} + \\ &\frac{n}{2\Delta t E_w} \{\sigma_w(t-\Delta t)\} - \{F_1(t)\}) \quad (35) \end{aligned}$$

在水体与河床、海床交界处, 基于力的平衡原则, 有水体的节点压力为

$$\sigma_{lq}(t) = \rho_w \ddot{\delta}_{lq}(t) = \rho_s \ddot{\delta}_u(t) \quad (36)$$

$\ddot{\delta}_u(t)$ 由式(31), (32) 求得。

式(35)求得孔压的发展, 便可计算有效应力大小, 它反映了土动力学中有效应力原理。

5 人工边界处理

人工边界区: 边界区在 x 和 y 方向的固相, 液相和孔压的表达式如下:

$$\begin{aligned} \delta_u(t+\Delta t, x+\Delta x) &= \delta_u(t, x) \\ \delta_u(t+\Delta t, y+\Delta y) &= \delta_u(t, y) \end{aligned} \quad (37)$$

$$\begin{aligned} \delta_v(t+\Delta t, x+\Delta x) &= \delta_v(t, x) \\ \delta_v(t+\Delta t, y+\Delta y) &= \delta_v(t, y) \end{aligned} \quad (38)$$

$$\begin{aligned} \sigma_w(t+\Delta t, x+\Delta x) &= \sigma_w(t, x) \\ \sigma_w(t+\Delta t, y+\Delta y) &= \sigma_w(t, y) \end{aligned} \quad (39)$$

自由表面按不透水的边界处理, $\tau = 0$ 。

6 算例

水深 20 m, 饱和砂土厚度为 20 m。图 1 为地震曲线, 图 2 为模型尺寸。饱和砂土平均剪切波速均为 275 m/s。土层重度为 19.2 kN/m³。土骨架回弹模量 $E_r = 4.375 \times 10^6$ Pa, 最大剪切模量 $G_{\max} = 2.0 \times 10^7$ Pa。地震波峰值加速度为 0.20 g。

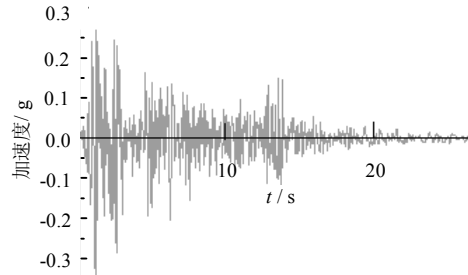


图 1 地震曲线

Fig.1 Curves of earthquake

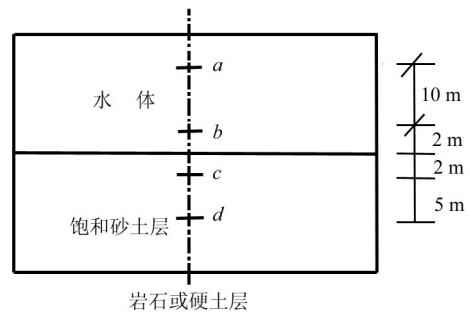


图 2 水体下地基示意图

Fig.2 Model of subsoil under water

为便于分析, 假设动水体压力比 R = 动水体压力/上覆水体重力; 土中动孔压比 r = 动孔压/上覆有效力, 图 3 中 c, d 点砂土中动孔压比接近很快接近 1, 表明河床或海床易液化, 图 4 中 b 点动水压力比说明强地震时水体可能受到压缩作用, 也许是一种位移波引起的, 在水-土交界面存在复杂效应, 于是出现了地震时有些河床下部水体泛滥现象, 同时水体压力波传播可能存在一定的限度, 这值得进一步研究。

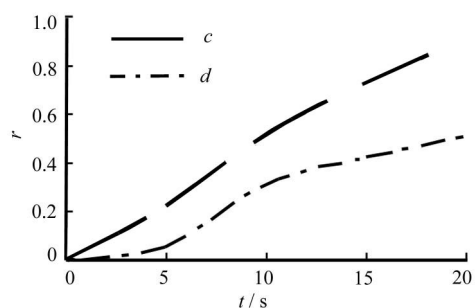


图 3 c, d 处土层孔压比

Fig.3 Pore- pressure- ratio at c, d

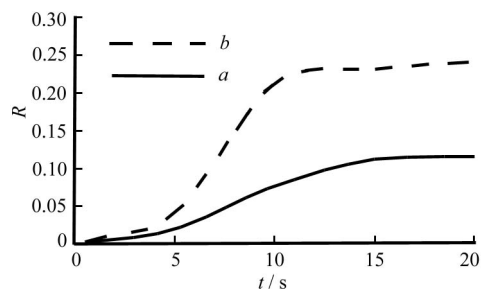


图 4 a, b 处动水体压力比
Fig.4 Water pressure-ratio at a, b

7 结 论

本文研究了用多相多孔介质动力学理论分析和模拟水下饱和土层波的传播问题涉及的运动方程和本构, 采用直接积分的差分法求解方程, 能够直接计算水体的压力、孔压, 为进一步分析水体下地层地震液化提供有力的分析手段。但问题较为复杂, 尚需进行大量的研究。

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